WYKŁAD WYDZIAŁOWY

w ramach seminarium

ARYTMETYCZNA GEOMETRIA ALGEBRAICZNA

(organizatorzy: Grzegorz Banaszak, Piotr Krasoń)

Czwartek **16 listopada 2017**, godz. **16:00**, sala **210**

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Local Constants for Galois Representations - Some Explicit Results

Ernst-Wilhelm Zink, joint work with Sazzad Ali Biswas

Streszczenie: Given a representation

$$\rho : Gal(K|F) \to GL_n(C) \tag{1}$$

of a Galois group of **number fields**, one may form the **augmented** Artin L-series $\Lambda(\chi_\rho, s)$ which is a meromorphic function of a complex variable s depending only on the character χ_ρ of that representation. As a classical result we have the functional equation

$$\Lambda(\chi_\rho, 1-s) = W(\chi_\rho) \cdot \Lambda(\overline{\chi_\rho}, s) \tag{2}$$

where $W(\chi_\rho)$ is a complex constant of absolute value 1, the **Artin root number**. (See for instance: An Introduction to the Langlands Program, p.81).

If $\rho = \chi_\rho) = \chi$ is 1-dimensional, in his derivation of the functional equation for Hecke $L-$series, **J.Tate** found a canonical decomposition of $W(\chi)$ into a product over all places ν of F :

$$W(\chi) = \prod_\nu W_\nu(\chi) \tag{3}$$

where the factors $W_\nu(\chi)$ = **local root numbers** depend only on the restriction of χ to the decomposition group $G_\nu = Gal(K_w|F_\nu)$ which comes as the Galois group of an **extension of local fields**.

Langlands (1970) noticed that also the higher-dimensional root numbers $W(\chi_\rho)$ of (2) should have a decomposition into local factors $W_\nu(\chi_\rho)$. The existence of these local root numbers has been proved by Langlands himself (in a unpublished preprint) and by Deligne, using global methods.

For a completely local existence proof one has to use a Brauer map $b_G : R_+(G) \to R(G), [H, \phi] \to \mathrm{Ind}_H^G(\phi)$, which realizes virtual representations of a (pro)finite group G in terms of 1-dimensional characters for subgroups H and to describe $\mathrm{Ker}(b_G)$ in terms of generating relations. Then it has to be verified that Tate's local root numbers $W_\nu(\chi)$ for 1-dimensional characters respect these generating relations.

In the talk we derive some explicit formulas for the local root numbers $W_\nu(\chi_\rho)$ if ρ is a Heisenberg representation and (following a paper of H.Koch) think on the role these formulas could play in a local existence proof.